Note on Queueing Game

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1 Unobservable Queues

2 Competition among Servers

Most of the models consider a game with two stages: servers act as leaders by announcing prices, and customers followed by selecting servers accordingly.

2.1 Heterogeneous Time Values

Suppose two identical exponential servers, i.e., $\mu_1 = \mu_2 = \mu$, competes via pricing decisions p_1 and p_2. The joint arrival process is Poisson with rate λ. Queues are unobservable, i.e., customers form their expectations for waiting time $W_i = \frac{1}{\mu_i - \lambda_i}$. Customers are heterogeneous among their time value c, which has a continuous CDF $F(\cdot)$. The value v of service is exogenously given and identical for all customers.

Note on *Servers may also make capacity decisions in the first stage, and time value may be discrete distributed.*

In the second stage, $p_i, i = 1, 2$ have been decided and assume that $p_1 \geq p_2$ WLOG. According to the utility maximizatoin problem, customers join queue 2 and 1 if $c < \frac{p_1 - p_2}{W_2 - W_1}$ and $\frac{p_1 - p_2}{W_2 - W_1} < c < \frac{v - p_1}{W_1}$, and balk if $c > \frac{v - p_1}{W_1}$. In other wrods,

$$\lambda_1^* = \lambda \left[F\left(\frac{v - p_1}{W_1} \right) - F\left(\frac{p_1 - p_2}{W_2 - W_1} \right) \right],$$
$$\lambda_2^* = \lambda F\left(\frac{p_1 - p_2}{W_2 - W_1} \right).$$

In the first stage, servers determines p_i to maximize their profits $p\lambda_i$.

Note on *The social welfare maximizing prices are lower than the equilibrium prices.*

2.2 Conditional Service Values $V(\lambda)$ on λ

Consider the competition between two identical M/G/1 servers of unobservable queues who serve customers with heterogeneous service values. All customers receive expected utility $V(\lambda)$, which is increasing concave, when the arrival rate is λ. Assume that $V'(0)$ is sufficiently large

to avoid trivialities. Obviously, the marginal utility $V'(\lambda^*)$ equals the expected full price in the equilibrium.

Bertrand Equilibrium (Price Competition).

Cournot Equilibrium (Rate Competition).

Monopoly Perspective.

Social Optimal Perspective.